Расширенная таблица интегралов

1. $\int \sin ax \cdot \sin bxdx = \dfrac{\sin(a-b)x}{2(a-b)} - \dfrac{\sin(a+b)x}{2(a+b)}, \quad a^2 \neq b^2.$

2. $\int \cos ax \cdot \cos bxdx = \dfrac{\sin(a-b)x}{2(a-b)} + \dfrac{\sin(a+b)x}{2(a+b)}, \; a^2 \neq b^2.$

3. $\int \sin ax \cdot \cos bxdx = -\dfrac{\cos(a+b)x}{2(a+b)} - \dfrac{\cos(a-b)x}{2(a-b)}, \; a^2 \neq b^2.$

4. $\int x \cdot \sin bx \cdot dx = \dfrac{1}{b^2} \cdot \sin bx - \dfrac{x}{b} \cdot \cos bx.$

5. $\int x^2 \cdot \sin bx \cdot dx = \dfrac{2x \cdot \sin bx}{b^2} - \dfrac{b^2x^2 - 2}{b^3} \cos bx.$

6. $\int x^3 \cdot \sin bx \cdot dx = \dfrac{3b^2x^2 - 6}{b^4} \sin bx + \dfrac{6x - b^2x^3}{b^3} \cos bx.$

7. $\int x \cdot \sin^2 bx \cdot dx = \dfrac{x^2}{4} - \dfrac{x \cdot \sin 2bx}{4b} - \dfrac{\cos 2bx}{8b^2}.$

8. $\int x \cdot \sin bx \cdot \sin cx \cdot dx = \dfrac{1}{2}\left[\dfrac{1}{(b-c)^2} \cdot \cos(b-c)x + \dfrac{x}{b-c} \sin(b-c)x - \dfrac{1}{(b+c)^2} \cos(b+c)x - \right.$

$\left. - \dfrac{x}{b+c} \sin(b+c)x \right], \; b^2 \neq c^2.$

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10. $\int x^2 \cos bx \cdot dx = \dfrac{2x \cdot \cos bx}{b^2} + \dfrac{b^2x^2 - 2}{b^3} \cdot \sin bx.$

11. $\int x^3 \cos bx \cdot dx = \dfrac{3b^2x^2 - 6}{b^4} \cos bx + \dfrac{b^2x^2 - 6x}{b^3} \cdot \sin bx.$

12. $\int x \cdot \cos^2 bxdx = \dfrac{x^2}{4} + \dfrac{x \cdot \sin 2bx}{48} + \dfrac{\cos 2bx}{8b^2}.$

13. $\int x \cos bx \cdot \cos cx \cdot dx = \dfrac{1}{2}\left[\dfrac{1}{(b-c)^2} \cos(b-c)x + \dfrac{x}{b-c} \sin(b-c)x + \right.$

$\left. + \dfrac{1}{(b+c)^2} \cos(b+c)x + \dfrac{x}{b+c} \sin(b+c)x \right], \; b^2 \neq c^2.$

14. $\int x \cdot \sin bx \cdot \cos cx \cdot dx = \dfrac{1}{2}\left[\dfrac{1}{(b+c)^2} \sin(b+c)x - \dfrac{x}{b+c} \cos(b+c)x + \right.$

$\left. + \dfrac{1}{(b-c)^2} \sin(b-c)x - \dfrac{x}{b-c} \cos(b-c)x \right], \; b^2 \neq c^2.$

15. $\int e^{ax}\cdot\sin bx\cdot dx=\frac{e^{ax}}{a^2+b^2}(a\sin bx-b\cos bx)$.

16. $\int e^{ax}\cdot\sin^2 bx\cdot dx=\frac{e^{ax}}{a^2+4b^2}(a\cdot\sin^2 bx-2b\cos bx\cdot\sin bx+\frac{2b^2}{a})=$

$=\frac{e^{ax}}{2a}-\frac{e^{ax}}{a^2+4b^2}(\frac{a}{2}\cos 2bx+b\sin 2bx)$.

17. $\int e^{ax}\cdot\sin bx\cdot\sin cx\cdot dx=\frac{e^{ax}}{2}\left[\frac{a\cdot\cos(b-c)x+(b-c)\cdot\sin(b-c)x}{a^2+(b-c)^2}-\right.$

$\left.-\frac{a\cdot\cos(b+)x+(b+c)\cdot\sin(b+c)x}{a^2+(b+c)^2}\right]$.

18. $\int e^{ax}\cdot\cos bx\cdot dx=\frac{e^{ax}}{a^2+b^2}(a\cos bx+b\sin bx)$.

19. $\int e^{ax}\cdot\cos^2 bx\cdot dx=\frac{e^{ax}}{a^2+4b^2}(a\cos^2 bx-2b\cdot\cos bx\cdot\sin bx+\frac{3b^2}{a})=$

$=\frac{e^{ax}}{2a}+\frac{e^{ax}}{a^2+4b^2}(\frac{a}{2}\cos 2bx+b\sin 2bx)$.

20. $\int e^{ax}\cdot\cos bx\cdot\cos cx\cdot dx=\frac{e^{ax}}{2}\left[\frac{ac0s(b+c)x+(b+c)\sin(b+c)x}{a^2+(b+c)^2}+\right.$

$\left.+\frac{a\cos(b-c)x+(b-c)\sin(b-c)x}{a^2+(b-c)^2}\right]$.

21. $\int e^{ax}\cdot\sin bx\cdot\cos cx\cdot dx=\frac{e^{ax}}{2}\left[\frac{a\sin(b+c)x-(b+c)\cos(b+c)x}{a^2+(b+c)^2}+\right.$

$\left.+\frac{a\sin(b-c)x-(b-c)\cos(b-c)x}{a^2+(b-c)^2}\right]$.

22. $\int e^{ax}\cdot\sin bx\cdot\cos bx\cdot dx=\frac{e^{ax}}{2a^2+8b^2}[a\cdot\sin 2bx-2b\cos 2bx]$.

23. $\int \operatorname{sh} ax\cdot\sin bx\cdot dx=\frac{a}{a^2+b^2}\operatorname{ch} ax\cdot\sin bx-\frac{b}{a^2+b^2}\operatorname{sh} ax\cdot\cos bx$.

24. $\int \operatorname{sh} ax\cdot\cos bx\cdot dx=\frac{a}{a^2+b^2}\operatorname{ch} ax\cdot\cos bx-\frac{b}{a^2+b^2}\operatorname{sh} ax\cdot\sin bx$.

25. $\int \operatorname{ch} ax\cdot\sin bx\cdot dx=\frac{a}{a^2+b^2}\operatorname{sh} ax\cdot\sin bx-\frac{b}{a^2+b^2}\operatorname{ch} ax\cdot\cos bx$.

26. $\int \operatorname{ch} ax\cdot\cos bx\cdot dx=\frac{a}{a^2+b^2}\operatorname{sh} ax\cdot\cos bx-\frac{b}{a^2+b^2}\operatorname{ch} ax\cdot\sin bx$.

27. $\int x\cdot e^{ax}\cdot dx=e^{ax}\left(\frac{x}{a}-\frac{1}{a^2}\right)$.

28. $\int x^2 e^{ax} dx = e^{ax}\left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right) = \frac{a^2x^2 - 2ax + 2}{a^3} \cdot e^{ax}$.

29. $\int x^3 e^{ax} \cdot dx = e^{ax}\left(\frac{x^3}{a} - \frac{3x^2}{a^2} + \frac{6x}{a^3}\right) = \frac{a^3x^3 - 3a^2x^2 + 6ax - 6}{a^4} \cdot e^{ax}$.

30. $\int x \cdot e^{ax} \cdot \sin bx \cdot dx = \frac{e^{ax}}{a^2+b^2}\left[\left(ax - \frac{a^2-b^2}{a^2+b^2}\right) \cdot \sin bx - \left(bx - \frac{2ab}{a^2+b^2}\right)\cos bx\right]$.

31. $\int x \cdot e^{ax} \cdot \cos bx \cdot dx = \frac{e^{ax}}{a^2+b^2}\left[\left(ax - \frac{a^2-b^2}{a^2+b^2}\right)\cos bx + \left(bx - \frac{2ab}{a^2+b^2}\right)\sin bx\right]$.

32. $\int\limits_0^\infty \frac{\cos bx}{a^2+x^2} dx = \frac{\pi}{2a} \cdot e^{-ab}$, $[a > 0, b \geq 0]$; $[\operatorname{Re} a > 0]$.

33. $\int\limits_0^\infty \frac{x \cdot \sin bx}{a^2+x^2} dx = \frac{\pi}{2} \cdot e^{-ab}$, $[a \geq 0, b \geq 0]$; $[\operatorname{Re} a > 0]$.

34. $\int\limits_{-\infty}^{\infty} \frac{e^{-bxi}}{a^2+x^2} dx = \frac{\pi}{a} \cdot e^{-|ab|}$, $[a > 0]$.

35. $\int\limits_0^\infty \frac{\sin bx}{x} dx = \frac{\pi}{2},\ 0,\ -\frac{\pi}{2}$, если $b > 0,\ b = 0,\ b < 0$.

36. $\int \sin ax \cdot \cos(bx+c) \cdot dx = -\frac{\cos(ax+bx+c)}{2(a+b)} - \frac{\cos(ax-bx-c)}{2(a-b)}$; $a \neq \pm b$; $a^2 \neq b^2$.

37. $\int \cos ax \cdot \cos(bx+c) \cdot dx = \frac{\sin(ax-bx-c)}{2(a-b)} + \frac{\sin(ax+bx+c)}{2(a+b)}$; $a \neq \pm b$; $a^2 \neq b^2$.

38. $\int\limits_0^\infty \frac{x \cdot \sin bx}{(x^2+a^2)^2} dx = \frac{\pi b}{4a} \cdot e^{-ab}$, $[a > 0, b > 0]$.

39. $\int\limits_0^\infty \frac{\sin^2 bx}{x^2} dx = \frac{\pi}{2} \cdot |b|$.

40. $\int x^2 \cdot \sin^2 bx \cdot dx = \frac{x^3}{6} - \left(\frac{x^2}{4b} - \frac{1}{8b^3}\right)\sin 2bx - \frac{x}{4b^2}\cos 2bx$.

41. $\int x^2 \cdot \cos^2 bx \cdot dx = \frac{x^3}{6} + \left(\frac{x^2}{4b} - \frac{1}{8b^3}\right)\sin 2bx - \frac{x}{4b^2}\cos 2bx$.

42. $\int x^4 \cdot \sin bx \cdot dx = \frac{1}{b^4}(4b^2x^3 - 24x)\sin bx - \frac{1}{b^5}(b^4x^4 - 12b^2x^2 + 24)\cos bx$.

43. $\int x^4 \cdot \cos bx \cdot dx = \frac{1}{b^4}(4b^2x^3 - 24x)\cos bx + \frac{1}{b^5}(b^4x^4 - 12b^2x^2 + 24)\sin bx$.

44. $\int \sin^3 3x \cdot \sin x \cdot dx = \frac{1}{320}[60\sin 2x - 30\sin 4x - 5\sin 8x + 4\sin 10x]$.

45. $\int \cos^3 3x \cdot \cos x \cdot dx = \frac{1}{320}[60\sin 2x + 30\sin 4x + 5\sin 8x + 4\sin 10x]$.

46. $\int \sin^3 3x \cdot \cos x \cdot dx = \frac{1}{320}[-60\cos 2x - 30\cos 4x + 5\sin 8x + 4\sin 10x] =$

$= \frac{27}{4}\sin^4 x - 18\sin^6 x + 18\sin^8 x - \frac{32}{5}\sin^{10x} = \frac{1}{160}(3-4\sin^2 x)^5 - \frac{3}{128}(3-4\sin^2 x)^4$.

47. $\int \cos^3 3x \cdot \sin x \cdot dx = \frac{1}{320}[60\cos 2x - 30\cos 4x + 5\cos 8x - 4\cos 10x] =$

$= \frac{27}{4}\cos^4 x - 18\cos^6 x + 18\cos^8 x - \frac{32}{5}\cos^{10x} = \frac{1}{160}(3-4\cos^2 x)^5 - \frac{3}{128}(3-4\cos^2 x)^4$.

48. $\int x \cdot e^{ax} \cdot \sin(bx+c) \cdot dx = \frac{x \cdot e^{ax}}{\sqrt{a^2+b^2}} \cdot \sin(bx+c+\varphi) - \frac{e^{ax}}{a^2+b^2} \cdot \sin(bx+c+2\varphi)$,

$\sin\varphi = -\frac{b}{\sqrt{a^2+b^2}}$, $\cos\varphi = \frac{a}{\sqrt{a^2+b^2}}$.

49. $\int x \cdot e^{ax} \cdot \cos(bx+c) \cdot dx = \frac{x \cdot e^{ax}}{\sqrt{a^2+b^2}} \cdot \cos(bx+c+\varphi) - \frac{e^{ax}}{a^2+b^2} \cdot \cos(bx+c+2\varphi)$.

50. $\int x \cdot \operatorname{sh} x \cdot dx = \frac{x}{a}\operatorname{ch} ax - \frac{1}{a^2}\operatorname{sh} ax$.

51. $\int x^2 \cdot \operatorname{sh} x \cdot dx = \left(\frac{x^2}{a} + \frac{2}{a^3}\right)\operatorname{ch} ax - \frac{2x}{a^2}\operatorname{sh} ax$.

52. $\int x^3 \cdot \operatorname{sh} x \cdot dx = \left(x^3 + \frac{6}{a^2}\right)\operatorname{ch} ax - \left(\frac{3}{a}x^2 + \frac{6}{a^3}\right)\operatorname{sh} ax$.

53. $\int x \cdot \operatorname{ch} x \cdot dx = \frac{x}{a}\operatorname{sh} ax - \frac{1}{a^2}\operatorname{ch} ax$.

54. $\int x^2 \cdot \operatorname{ch} x \cdot dx = \left(\frac{x^2}{a} + \frac{2}{a^3}\right)\operatorname{sh} ax - \frac{2x}{a^2} \cdot x \cdot \operatorname{ch} ax$.

55. $\int x^3 \cdot \operatorname{ch} x \cdot dx = \left(x^3 + \frac{6}{a^2}\right)\operatorname{sh} ax - \left(\frac{3}{a}x^2 + \frac{6}{a^3}\right)\operatorname{ch} ax$.